Problem Set

MA17Q4-C

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[1] Basic properties of growth rates.

Romer 4e, Problem 1.1. The growth rate of a variable equals the time derivative of its log, i.e. $\dot{X}(t)/X(t) = \frac{d}{dt}[\ln X(t)]$, where $\dot{X}(t) = \frac{dX}{dt}(t)$. Use this fact to show:

(a) If $Z(t) = X(t)Y(t)$, then $\dot{Z}(t)/Z(t) = [\dot{X}(t)/X(t)] + [\dot{Y}(t)/Y(t)]$.
(b) If $Z(t) = X(t)/Y(t)$, then $\dot{Z}(t)/Z(t) = [\dot{X}(t)/X(t)] - [\dot{Y}(t)/Y(t)]$.
(c) If $Z(t) = X(t)^{\alpha}$, then $\dot{Z}(t)/Z(t) = \alpha\dot{X}(t)/X(t)$.

[2] Prediction for the near future

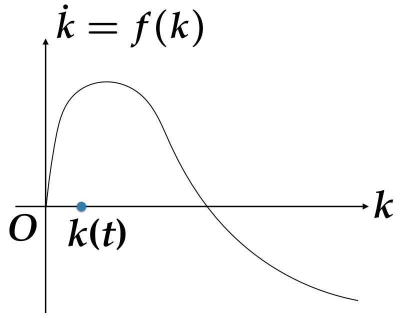
Suppose that your clock shows it is time t now. Suppose also that some important variable $k(t)$ satisfies a differential equation $\dot{k} = f(k)$, where $\dot{k}$ denotes the time derivative of k, f is a function of k. You are interested in prediction of $k(t + \Delta t)$ for $\Delta t > 0$.

In the cases listed below, is $k(t + \Delta t)$ greater than, smaller than or equal to $k(t)$?

(a) $f(k(t)) > 0$
(b) $f(k(t)) < 0$
(c) $f(k(t)) = 0$

[3] Prediction for the distant future

The below figure sketches the graph of f. Describe what will happen to $k(t)$ as $t \to \infty$, starting from $k(t)$ indicated by the dot.



Notation

log or ln

Both log and ln denote the natural logarithm. It is the inverse function of e^x, where $e \simeq 2.718281...$ is Napier's constant; i.e., $e^{\ln x} = x$, and $\ln e^x = x$. When we want to specify the base b of log, we explicitly write it. For instance, the common logarithm is denoted by $\log_{10} y$.

We will use the following formulas very often: for $x, y > 0$,

$$\ln xy = \ln x + \ln y, \quad \ln \frac{x}{y} = \ln x - \ln y, \quad \ln x^\alpha = \alpha \ln x.$$

Derivatives

Suppose a variable x changes its values with time. Since we usually use letter t for time, we write $x(t)$ to show it is time dependent; it is a function of time. We sometimes don't bother to write t when the time-dependence is obvious from the context. The derivative of x with respect to time

$$\frac{dx}{dt} = \lim_{\Delta t \to 0} \frac{x(t + \Delta t) - x(t)}{(t + \Delta t) - t}$$

is denoted by $\dot{x}(t)$. $\dot{x}$ is voiced "x dot."

Let f be a function of x. The derivative of $f(x)$ with respect to x is denoted by $f'(x)$ (f' is voiced "f prime"). When f is a function of x and x is a function of time, the most unambiguous expression, $f(x(t))$, is sometimes written simply as $f(x)$. Because $f(x)$ is a function of time (through the time dependence of x), we can take time-drivative of $f(x)$, which is

$$\frac{df(x)}{dt} = f'(x(t))\dot{x}(t).$$

For example, let $f = \ln$. Recall that $f'(x) = \ln'(x) = \frac{1}{x}$. Thus,

$$\frac{d}{dt}(\ln x(t)) = \frac{\dot{x}(t)}{x(t)}.$$

Growth Rates

Since we will study economic growth, we will analyze the rates of growth of many economic variables. Mathematically, the growth rate of x is defined by $\frac{\dot{x}}{x}$, which will be denoted by g_x in this class (this is not standard). Recall that

$$\frac{\dot{x}}{x} \simeq \frac{x(t + \Delta t) - x(t)}{\Delta t \cdot x(t)} = \frac{1}{\Delta t}\left[\frac{x(t + \Delta t) - x(t)}{x(t)}\right].$$

By multiplying $\frac{1}{\Delta t}$ and the instantaneous rate of change, $\frac{x(t+\Delta t) - x(t)}{x(t)}$, the latter is translated up into the rate of change in a unit length of time, a year, quarter, or month for instance.

Answer sheet. Please write your name and id number.