Macroeconomics 17q2
Document No. ma17q2-G

Problem Set

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June 29, 2017

Augmented Solow model by Mankiw, Romer and Weil (1992)

Let $0 < \alpha, \beta < 1$ and $0 < \alpha + \beta < 1$. Consider the following production function,

$$Y = F(K, H, AL) = K^\alpha H^\beta (AL)^{1-\alpha-\beta}.$$

With constant saving rates s_k and s_h for, respectively, physical and human capital, and with the depreciation rate δ common to both capital, the capital accumulation is described by the following two equation.

$$\dot{K} = s_k Y - \delta K$$
$$\dot{H} = s_h Y - \delta H$$

(1) Show that $k := K/(AL)$ and $h := H/(AL)$ satisfy

$$\dot{k} = s_k f(k, h) - (\delta + g + n)k$$
$$\dot{h} = s_h f(k, h) - (\delta + g + n)h,$$

where

$$f(k, h) = k^\alpha h^\beta = \frac{Y}{AL} = \frac{F(K, H, AL)}{AL}.$$

(2) Compute the steady state (k^*, h^*).

(3) Verify that, on the balanced growth path,

$$\ln\left(\frac{Y}{L}\right) = \ln A(0) + gt + \frac{\alpha}{1-\alpha-\beta}\ln(s_k) + \frac{\beta}{1-\alpha-\beta}\ln(s_h) - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(\delta + g + n).$$

(4) Verify that, on the balanced growth path,

$$\ln\left(\frac{Y}{L}\right) = \ln A(0) + gt + \frac{\alpha}{1-\alpha}\ln(s_k) - \frac{\alpha}{1-\alpha}\ln(\delta + g + n) + \frac{\beta}{1-\alpha}\ln(h^*).$$

Answer sheet. Please write your name and id number.